

Pion Interactions in Chiral Field Theories*

M.D. Scadron

Physics Dept. Univ. of Arizona, Tucson AZ 85721, USA

(October 7, 1999)

We study in various chiral models the pion charge radius, π_{e3} form factor ratio, $\pi^0 \rightarrow \gamma\gamma$ amplitude, charge pion polarizabilities, $\gamma\gamma \rightarrow \pi^0\pi^0$ amplitude at low energies and the $\pi\pi$ s-wave $I = 0$ scattering length. We find that a quark-level linear sigma approach (also being consistent with tree-level vector meson dominance) is quite compatible with all of the above data.

PACs numbers: 11.30.Rd, 11.40.Ha, 12.40.Vv

I. INTRODUCTION

In this paper we study the interactions of observed pions with inferred scalar σ meson [1,2] and fermion quark SU(2) fields in a chiral-invariant manner at low energies. Specifically we consider two chiral theories:

a) A chiral quark model (CQM) dynamically inducing [2] the entire quark-level SU(2) linear σ model (L σ M) but depending on no free parameters.

b) Chiral perturbation theory (ChPT) involving ten strong interaction parameters $L_1 - L_{10}$ [3-5], now called low energy constants (LECs).

Following the surveys of Donoghue and Holstein [6,7], we compare the predictions of the above two theories with the measured values of the i) pion charge radius, ii) π_{e3} form factor ratio F_A/F_V at zero invariant momentum transfer and the $\pi^0 \rightarrow \gamma\gamma$ amplitude, iii) charged pion polarizabilities, iv) $\gamma\gamma \rightarrow \pi^0\pi^0$ amplitude at low energies, v) $\pi\pi$ s-wave $I = 0$ scattering length.

We begin in Sec.II with the quark-level Goldberger-Treiman relation (GTR), (its meson analog) the KSRF relation [8] and the link to vector meson dominance (VMD). Then in Sec.III we examine the pion charge radius r_π in the above two chiral theories. Next in Sec.IV, we first review $\pi^0 \rightarrow \gamma\gamma$ decay and then study its isospin-rotated semileptonic weak analog $\pi^+ \rightarrow e^+\nu\gamma$, giving rise to the form factor ratio $F_A/F_V \equiv \gamma$ at zero invariant momentum transfer. This naturally leads in Sec.V to the charged pion electric polarizability α_{π^+} due to the model-independent relation [9,6] between α_{π^+} and the above π_{e3} ratio γ . Finally in Sec.VI we review the Weinberg soft-pion prediction [10] for the s-wave $I = 0$ $\pi\pi$ scattering length and its chiral-breaking corrections.

In all of the above cases the predictions of the CQM-L σ M and ChPT chiral theories are compared with the measured values of $r_\pi, \gamma, \alpha_{\pi^+}, a_{\pi\pi}^{(0)}$. We review these results in Sec.VII.

II. CQM LINK TO GTR, VMD AND KSRF

The chiral quark model (CQM) involves u and d quark loops coupling in a chiral invariant manner to external pseudoscalar pions (and scalar σ mesons). In order to manifest the Nambu-Goldstone theorem with $m_\pi = 0$ and conserved axial currents $\partial A^\pi = 0$, it is clear that the quark-level Goldberger-Treiman relation (GTR) must hold:

$$f_\pi g_{\pi qq} = m_q. \quad (1)$$

Here the pion decay constant is $f_\pi \approx 90 \text{ MeV}$ in the chiral limit [11] and the constituent quark mass is expected to be $m_q \sim m_N/3 \sim 320 \text{ MeV}$. Indeed, this dynamical quark mass $m_q \sim 320 \text{ MeV}$ also follows from nonperturbative QCD considerations [12], scaled to the quark condensate.

Given these nonperturbative mass scales of 90 MeV and 320 MeV, the dimensionless pion-quark coupling should be $g_{\pi qq} \sim 320/90 \approx 3.6$. The latter scale of 3.6 also follows from the phenomenological πNN coupling constant [13] $g_{\pi NN} \approx 13.4$ since then

$$g_{\pi qq} = g_{\pi NN}/3g_A \approx 3.5 \quad (2)$$

*Published in *Mod. Phys. Lett.* **A14** (1999) 1349.

for the measured value [14] $g_A \approx 1.267$. In fact in the SU(2) CQM with u and d loops for $N_c = 3$, cutoff-independent dimensional regularization dynamically generates the entire quark-level linear sigma model (L σ M) and also requires [2]

$$g_{\pi qq} = 2\pi/\sqrt{3} \approx 3.6276 \quad \text{and} \quad m_\sigma = 2m_q. \quad (3)$$

The former coupling is compatible with (1) and (2) and the latter scalar-mass relation also holds in the four-quark chiral NJL scheme [15] in the chiral limit. If one substitutes $g_{\pi qq} = 2\pi/\sqrt{3}$ back into the GTR (1), one finds

$$m_q = f_\pi 2\pi/\sqrt{3} \approx 325 \text{ MeV} \quad \text{and} \quad m_\sigma = 2m_q \approx 650 \text{ MeV}. \quad (4)$$

Moreover the CQM quark loop for the vacuum to pion matrix element of the axial current $\langle 0 | \bar{q} \frac{1}{2} \lambda_3 \gamma_\mu \gamma_5 q | \pi^\circ \rangle = i f_\pi q_\mu$ as depicted in Fig.1, generates the log-divergent gap equation in the chiral-limit once the GTR (1) is employed:

$$1 = -i4N_c g_{\pi qq}^2 \int_0^\Lambda \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - m_q^2)^2}. \quad (5)$$

Given the pion-quark coupling in (2) or (3), it is easy to show that the cutoff in (5) must be $\Lambda \approx 2.3 m_q \approx 750 \text{ MeV}$. This naturally separates the “elementary” σ with $m_\sigma \approx 650 \text{ MeV}$ in (4) from the “bound state” ρ meson with $m_\rho \approx 770 \text{ MeV}$.

In fact it was shown in the third reference in [1] that the CQM u and d quark loops of Fig 2 for $\rho^\circ \rightarrow \pi^+ \pi^-$ lead to the chiral-limiting relation

$$g_{\rho\pi\pi} = g_\rho \left[-i4N_c g_{\pi qq}^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - m_q^2)^2} \right] = g_\rho, \quad (6)$$

where the gap equation (5) is used. Experimentally [14] $g_{\rho\pi\pi}^2/4\pi \approx 3.0$ or $|g_{\rho\pi\pi}| \approx 6.1$, while the rho-quark coupling measured in $\rho^\circ \rightarrow e^+ e^-$ is $|g_\rho| \approx 5.0$. From the perspective of vector meson dominance (VMD), equ. (6) is the well-known VMD universality relation [16]. Moreover CQM quark loops with an external ρ° replaced by a photon γ corresponds to the VMD $\rho^\circ - \gamma$ analogy [17]. However from the perspective of the dynamical generated L σ M, $g_{\rho\pi\pi} = g_\rho$ in (6) corresponds to a $Z = 0$ compositeness condition [18]. It shrinks “loops to trees”, implying that the L σ M analogue equation $g_{\sigma\pi\pi} = g'$ can treat the σ as an elementary particle while the NJL model can treat the σ as a $\bar{q}q$ bound state.

Lastly, the meson analogue of the fermion GTR (1) is the KSRF relation [8], generating the ρ mass as

$$m_\rho \approx \sqrt{2} f_\pi (g_\rho g_{\rho\pi\pi})^{1/2} \approx 730 \text{ MeV}. \quad (7)$$

We recall that (7) also follows by equating the $I = 1$ πN VMD ρ -dominated amplitude $g_{\rho\pi\pi} g_\rho / m_\rho^2$ to the chiral-symmetric current algebra amplitude $1/2 f_\pi^2$ [19]. In short, the CQM quark loops combined with the quark-level GTR (1) dynamically generate the entire L σ M and the NJL relation (3), along with the VMD universality and KSRF relations (6) and (7). This collective CQM-L σ M-NJL-VMD-KSRF picture [20] will represent our first chiral approach to pion interactions as characterized by $r_\pi, F_A/F_V, \alpha_{\pi^+}$ and $a_{\pi\pi}^{(0)}$.

III. PION CHARGE RADIUS

It is now well-understood [21] that the CQM quark loop-depicted in Fig 3 generates the pion charge radius (squared) for $N_c = 3$ in the chiral limit with $f_\pi \approx 90 \text{ MeV}$ as

$$\langle r_\pi^2 \rangle = \frac{3}{4\pi^2 f_\pi^2} \approx (0.60 fm)^2. \quad (8)$$

Stated another way, using the CQM-L σ M $g_{\pi qq} = 2\pi/\sqrt{3}$ coupling relation in (3), r_π in (8) can be expressed in terms of the GTR as the inverse Compton mass

$$r_\pi = \frac{\sqrt{3}}{2\pi f_\pi} = \frac{1}{g_{\pi qq} f_\pi} = \frac{1}{m_q} \approx 0.61 \text{ fm}, \quad (9)$$

using the quark mass scale in (4). In either case this predicted pion charge radius is quite close to the measured value [22] of 0.63 fm. A CQM interpretation of (9) is that the quarks in a Goldstone $\bar{q}q$ pion are tightly bound and *fuse* together, so that $m_\pi = 0$ in the chiral limit with pion charge radius $r_\pi = 1/m_q$ the size of just *one* quark.

Another link of r_π to the CQM-L σ M-VMD-KSRF picture derives from examining the standard VMD result

$$r_\pi = \frac{\sqrt{6}}{m_\rho} \approx 0.63 \text{ fm}. \quad (10)$$

Not only is (10) in agreement with experiment, but equating the square root of (8) to (10) and invoking the KSRF relation (7) in turn requires with $g_{\rho\pi\pi} = g_\rho$,

$$g_{\rho\pi\pi} = 2\pi \approx 6.28. \quad (11)$$

This relation has long been stressed in a L σ M context [23], and is of course compatible with the measured $\rho \rightarrow 2\pi$ coupling $|g_{\rho\pi\pi}| \approx 6.1$.

But a deeper CQM-L σ M connection exists due to (11). In ref [2] the CQM quark loops of Fig 4 for the vacuum to ρ° -matrix element $\langle 0|V_\mu^{em}|\rho^\circ\rangle = (em_\rho^2/g_\rho)\varepsilon_\mu$ was shown to dynamically generate the vector polarization function $\Pi(k^2, m_q)$ in the chiral limit $k^2 \rightarrow 0$,

$$\frac{1}{g_\rho^2} = \Pi(k^2 = 0, m_q) = -\frac{8iN_c}{6} \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m_q^2)^2} = \frac{1}{3g_{\pi qq}^2}, \quad (12)$$

by use of the gap equation (5). Then invoking the CQM-L σ M coupling $g_{\pi qq} = 2\pi/\sqrt{3}$ from (3), equation (12) together with the VMD relation (6) leads to

$$g_\rho = g_{\rho\pi\pi} = \sqrt{3}g_{\pi qq} = 2\pi, \quad (13)$$

which recovers (11).

The second chiral approach, referred to as chiral perturbation theory (ChPT), is not considered as a model but a method relating various chiral observables. However the cornerstone of ChPT is that the pion charge radius r_π *diverges* [24] in the chiral limit (CL) and that away from the CL r_π is fixed by the LEC L_9 as

$$\langle r_\pi^2 \rangle = 12 L_9 / f_\pi^2 + \text{chiral loops}. \quad (14)$$

To the extent that L_9 is scaled to the VMD value of r_π in (10) and the chiral loops in (14) are small [24], this ChPT-VMD approach leads to reasonable phenomenology, as emphasized in ref. [6]. But from our perspective, this ChPT relation (14) circumvents the physics of (8)-(13). Instead the CL r_π is *finite* and is 0.60-0.61 fm in (8) or (9), near the measured value 0.63 ± 0.01 fm. The LEC L_9 does not explain this fact.

IV. Π_{E3}^+ FORM FACTORS AND $\pi^\circ \rightarrow 2\gamma$ DECAY

The CQM u and d quark loops for $\pi^\circ \rightarrow 2\gamma$ decay in Fig 5 generate the Steinberger-ABJ anomaly amplitude [25] $F_{\pi^\circ\gamma\gamma}\varepsilon_{\mu\nu\alpha\beta}(\varepsilon^{*\mu}\varepsilon^\nu k'k)$ where

$$|F_{\pi^\circ\gamma\gamma}| = \frac{\alpha}{\pi f_\pi} \approx 0.0258 \text{ GeV}^{-1} \quad (15)$$

in the $m_\pi = 0$ chiral limit, using the quark-level GTR (1). Since no pion loop can contribute to $\pi^\circ \rightarrow 2\gamma$, the CQM-Steinberger-ABJ anomaly result (15) is also the L σ M amplitude. Then with $m_q \approx 325$ MeV traversing the quark loops in Fig 5, the $\pi^\circ\gamma\gamma$ decay rate from (15) is predicted to be [26]

$$\Gamma_{\pi^\circ\gamma\gamma} = \frac{m_\pi^3}{64\pi} |F_{\pi^\circ\gamma\gamma}|^2 \approx 8 \text{ eV} \left[\frac{2m_q}{m_\pi} \sin^{-1} \left(\frac{m_\pi}{2m_q} \right) \right]^4 \approx 8 \text{ eV} \quad (16)$$

with $m_\pi/2m_q \approx 0.21 \ll 1$. Of course the latter rate in (16) is near the observed value [14] $(7.74 \pm 0.6) \text{ eV}$.

Treating $\pi^+ \rightarrow e^+\nu\gamma$ as an off-shell version of $\pi^\circ \rightarrow \gamma\gamma$ decay, the CVC SU(2) rotation of (15) predicts the zero momentum transfer vector form factor [27]

$$F_V(0) = \frac{\sqrt{2}}{8\pi^2 f_\pi} \sim 0.19 \text{ GeV}^{-1} \sim 0.027 m_\pi^{-1}. \quad (17)$$

A pure quark model is then in doubt [28], because the analogue axial vector quark loop is identical to (17) so that $\gamma_{qk} = F_A(0)/F_V(0)|_{qk} = 1$, which is about twice the observed γ . In fact the 1998 PDG values [14], statistically dominated by the same experiment (minimizing the systematic errors) gives

$$\gamma_{exp} = \frac{F_A(0)}{F_V(0)} = \frac{0.0116 \pm 0.0016}{0.017 \pm 0.008} = 0.68 \pm 0.34. \quad (18)$$

However the $L\sigma M$ generates both quark and meson loops to the $\pi^+ \rightarrow e^+ \nu \gamma$ amplitude as depicted in Fig 6. This leads to the $F_A(0)$ axial current form factor [29]

$$F_A(0) = F_A^{qk}(0) + F_A^{meson}(0) = \sqrt{2}(8\pi^2 f_\pi)^{-1} - \sqrt{2}(24\pi^2 f_\pi)^{-1} = \sqrt{2}(12\pi^2 f_\pi)^{-1}, \quad (19)$$

or with a γ found from (19) divided by (17):

$$\gamma_{L\sigma M} = \frac{F_A(0)}{F_V(0)} = 1 - \frac{1}{3} = \frac{2}{3}. \quad (20)$$

It is satisfying that $\gamma_{L\sigma M}$ in (20) accurately reflects the central value of the observed ratio in (18).

On the other hand, the ChPT picture appears [6] to give values of $\gamma = F_A(0)/F_V(0)$ varying from 0 (in leading-log approximation) to 1 in a chiral quark model-type calculation [6]

$$\frac{F_A(0)}{F_V(0)} = 32\pi^2(L_9 + L_{10}) = 1. \quad (21)$$

The latter (incorrect) value holds when the pion charge radius is (correctly) given by the CQM-VMD value [6]

$$\langle r_\pi^2 \rangle = \frac{12L_9}{f_\pi^2} = \frac{3}{4\pi^2 f_\pi^2}. \quad (22)$$

V. CHARGED PION POLARIZABILITIES AND $\gamma\gamma \rightarrow \pi\pi$ SCATTERING

Electric and magnetic polarizabilities characterize the next-to-leading order (non-pole) terms in a low energy expansion of the $\gamma\pi \rightarrow \gamma\pi$ amplitude. Although in rationalized units (with $\alpha = e^2/4\pi \approx 1/137$) the classical energy U generated by electric and magnetic fields is $U = (\frac{1}{2}) \int d^3x (\vec{E}^2 + \vec{B}^2)$, we follow recent convention and define charged or neutral electric (α_π) and magnetic (β_π) polarizabilities from the effective potential V_{eff} as

$$V_{eff} = -\frac{4\pi}{2}(\alpha_\pi \vec{E}^2 + \beta_\pi \vec{B}^2). \quad (23)$$

With this definition [30], α_π and β_π have units of volume expressed in terms of $10^{-4} fm^3 = 10^{-43} cm^3$. Chiral symmetry with $m_\pi \rightarrow 0$ requires $\alpha_\pi + \beta_\pi \rightarrow 0$ for charged or neutral pion polarizabilities and this appears to be approximately borne out by experiment. As for the charged pion polarizabilities, three different experiments for $\gamma\pi^+ \rightarrow \gamma\pi^+$ respectively yield the values [31-33]

$$\alpha_{\pi^+} = (6.8 \pm 1.4 \pm 1.2) \times 10^{-4} fm^3 \quad (24)$$

$$\alpha_{\pi^+} = (20 \pm 12) \times 10^{-4} fm^3 \quad (25)$$

$$\alpha_{\pi^+} = (2.2 \pm 1.6) \times 10^{-4} fm^3. \quad (26)$$

In the CQM- $L\sigma M$ scheme, the simplest way to find the charged pion electric polarizability α_{π^+} is to link it to the π_{e3} ratio $\gamma = F_A(0)/F_V(0)$ via the model-independent relation

$$\alpha_{\pi^+} = \frac{\alpha}{8\pi^2 m_\pi f_\pi^2} \gamma \quad (27)$$

first derived by Terent'ev [9]. Since one knows that $\gamma_{L\sigma M} = 2/3$ from (20) (consistent with observation), the $L\sigma M$ combined with (27) predicts

$$\alpha_{\pi^+}^{L\sigma M} = \frac{\alpha}{12\pi^2 m_\pi f_\pi^2} \approx 3.9 \times 10^{-4} fm^3. \quad (28a)$$

This $L\sigma M$ polarizability (28a) is internally consistent because a direct (but tedious) calculation of α_{π^+} due to quark loops and meson loops gives the $L\sigma M$ value [34]

$$\alpha_{\pi^+}^{L\sigma M} = \alpha_{\pi^+}^{qk} + \alpha_{\pi^+}^{meson} = \frac{\alpha}{8\pi^2 m_\pi f_\pi^2} - \frac{\alpha}{24\pi^2 m_\pi f_\pi^2} = \frac{\alpha}{12\pi^2 m_\pi f_\pi^2}, \quad (28b)$$

in complete agreement with (28a). This $L\sigma M$ value for α_{π^+} is midway between the measurements in (24) and (26).

The recent phenomenological studies [35] of Kaloshin and Serebryakov (KS) analyze the Mark II data [36] for $\gamma\gamma \rightarrow \pi^+\pi^-$ and find $(\alpha - \beta)_{\pi^+} = (4.8 \pm 1.0) \times 10^{-4} \text{ fm}^3$ and $(\alpha + \beta)_{\pi^+} = (0.22 \pm 0.06) \times 10^{-4} \text{ fm}^3$. These results correspond to

$$\alpha_{\pi^+}^{KS} = (2.5 \pm 0.5) \times 10^{-4} \text{ fm}^3, \quad (29)$$

not too distant from the $L\sigma M$ value (28) and (26), but substantially below (24). However (29) is very close to the ChPT prediction of Donoghue and Holstein [6] (DH)

$$\alpha_{\pi^+}^{ChPT} = \left(\frac{4\alpha}{m_\pi f_\pi^2}\right)(L_9 + L_{10}) \approx 2.8 \times 10^{-4} \text{ fm}^3, \quad (30)$$

if one uses the implied value of $L_9 + L_{10}$ from $\gamma \approx 0.5$. But in ref.[7] they show in Figs.7 and 9 that a full dispersive calculation for $\gamma\gamma \rightarrow \pi^+\pi^-$ (including the dominant pole term) reasonably maps out the low energy Mark II data from $0.3\text{GeV} < E < 0.7 \text{ GeV}$ for any π^+ polarizability in the range

$$1.4 \times 10^{-4} \text{ fm}^3 < \alpha_{\pi^+}^{DH} < 4.2 \times 10^{-4} \text{ fm}^3. \quad (31)$$

Moreover both data analyses in (24), (26) or in (29), (31) also surround the $L\sigma M$ -Terent'ev-L'vov prediction for α_{π^+} in (28), so the ChPT prediction (30) is not unambiguously “gold plated” as ChPT advocates maintain.

Next we study low energy $\gamma\gamma \rightarrow \pi^0\pi^0$ scattering, where there is no pole term and the polarizabilities α_{π^0} and β_{π^0} are much smaller than for charged pions. Even the sign of α_{π^0} is not uniquely determined. In Fig. 7a we display the comparison of the $\gamma\gamma \rightarrow \pi^0\pi^0$ cross section in the low energy region $0.3 \text{ GeV} < E < 0.7 \text{ GeV}$ as found from Crystal Ball data [37] and a parameter-independent dispersive calculation (solid line) [7, 38], verses the one-loop ChPT prediction (dashed line) [39]. This graph has already been displayed in refs [5,7]. As noted by Leutwyler [5], this first-order “gold-plated prediction of ChPT” might cause reason to panic. In fact, Kaloshin and Serebryakov in their Physics Letter Fig. 1 of ref. [35], now displayed as our Fig. 7b, show a solid line through their (gold-plated) $\gamma\gamma \rightarrow \pi^0\pi^0$ prediction [40] made five years *prior* to the Crystal Ball results. This was based in part upon the *existence of a broad scalar $\varepsilon(700)$* i.e. the $L\sigma M$ $\sigma(700)$. On the other hand, ChPT theory rules out [3, 20] the existence of an $\varepsilon(700)$ scalar. Stated in reverse, perhaps the ChPT rise of $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ above 10nb in the 700 MeV region (inconsistent with Crystal Ball data) could be corrected if the $\varepsilon(700)$ (or the $L\sigma M$ -NJL σ meson in eq. (3)) were taken into account.

To make this point in another way, recall that the decay $A_1 \rightarrow \pi(\pi\pi)_s \text{ wave}$ has a very small measured rate [14] $\Gamma = (1 \pm 1) \text{ MeV}$. This can be understood [41] in the context of our CQM- $L\sigma M$ picture giving rise to the *two* quark loop graphs in Fig 8. Owing to the general Dirac-matrix partial fraction identity

$$\frac{1}{\not{p} - m_q} 2m_q \frac{1}{\not{p} - m_q} = -\gamma_5 \frac{1}{\not{p} - m_q} - \frac{1}{\not{p} - m_q} \gamma_5, \quad (32)$$

there is a soft pion theorem (SPT) which forces the “box” and “triangle” quark loops in Fig 8 to interfere destructively. Specifically the quark-level GTR in (1) and (32) above give in the soft pion $p_\pi \rightarrow 0$ limit

$$\langle (\pi\pi)_{sw} \pi | A_1 \rangle = \left[-\frac{i}{f_\pi} \langle \sigma_\pi | A_1 \rangle + \frac{i}{f_\pi} \langle \sigma_\pi | A_1 \rangle \right] = 0, \quad (33)$$

in agreement with the data.

Applying a similar soft-pion argument to the two neutral pion quark loop graphs in Fig. 9 representing the CQM- $L\sigma M$ amplitude for $\gamma\gamma \rightarrow \pi^0\pi^0$ scattering, a quark box plus quark triangle cancellation due to the identity (32) leads to the SPT prediction

$$\langle \pi^0\pi^0 | \gamma\gamma \rangle \rightarrow \left[-\frac{i}{f_\pi} \langle \sigma | \gamma\gamma \rangle + \frac{i}{f_\pi} \langle \sigma | \gamma\gamma \rangle \right] = 0. \quad (34)$$

Qualitatively this “ σ interference” may be what ref. [40] predicts and what ChPT is lacking in the data plots of Fig. 3 and Fig. 2 in refs. [5,7] respectively, corresponding to our Fig. 7a.

VI. $\pi\pi$ S-WAVE $I = 0$ SCATTERING LENGTH

In the context of the CQM- $\mathcal{L}\sigma\mathcal{M}$ picture, $\pi\pi$ quark box graphs “shrink” back to “tree” diagrams due to the $Z = 0$ [18] structure of this theory [2]. Thus one need not go beyond the original tree-level $\mathcal{L}\sigma\mathcal{M}$ [42,43] as recently emphasized by Ko and Rudaz in ref. [1]. Following Weinberg’s [10] soft-pion expansion, Ko and Rudaz express the $\pi\pi$ scattering amplitude in ref. [1] as

$$M_{ab,cd} = A(s, t, u)\delta_{ab}\delta_{cd} + A(t, s, u)\delta_{ac}\delta_{bd} + A(u, t, s)\delta_{ad}\delta_{cb} \quad (35)$$

and write the s channel $I = 0$ amplitude as

$$T^0(s, t, u) = 3A(s, t, u) + A(t, s, u) + A(u, t, s). \quad (36)$$

Then they note that the original (tree-level) $\mathcal{L}\sigma\mathcal{M}$ predicts

$$A(s, t, u) = -2\lambda \left[1 - \frac{2\lambda f_\pi^2}{m_\sigma^2 - s} \right], \quad (37)$$

where away from the chiral limit $m_\pi \neq 0$ one knows

$$\lambda = \frac{g_{\sigma\pi\pi}}{f_\pi} = \frac{(m_\sigma^2 - m_\pi^2)}{2f_\pi^2}, \quad (38)$$

regardless of the value of m_σ [42-44]

Substituting (38) into (37) one obtains a slight modification of the Weinberg $(s - m_\pi^2)/f_\pi^2$ structure:

$$A(s, t, u) = \left(\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s} \right) \left(\frac{s - m_\pi^2}{f_\pi^2} \right), \quad (39)$$

so that the s-wave $I=0$ scattering length at $s = 4m_\pi^2$, $t = u = 0$ becomes in the $\mathcal{L}\sigma\mathcal{M}$ with $\varepsilon = m_\pi^2/m_\sigma^2 \approx 0.046$ from (3):

$$a_{\pi\pi}^{(0)}|_{\mathcal{L}\sigma\mathcal{M}} \approx \left(\frac{7 + \varepsilon}{1 - 4\varepsilon} \right) \frac{m_\pi}{32\pi f_\pi^2} \approx (1.23) \frac{7m_\pi}{32\pi f_\pi^2} \approx 0.20 m_\pi^{-1}. \quad (40)$$

This 23% enhancement of the Weinberg prediction of [10] $0.16 m_\pi^{-1}$ is also obtained from ChPT considerations [45]. It is interesting that ChPT simulates [46] the 23% enhancement found from the $\mathcal{L}\sigma\mathcal{M}$ analysis in (37)-(40) above, especially in light of the “miraculous” cancellation of $\mathcal{L}\sigma\mathcal{M}$ tree level terms, as explicitly shown in eqs. (5.61)-(5.62) of ref. [43] Such a $\mathcal{L}\sigma\mathcal{M}$ -induced cancellation instead resembles the SPT eq. (34) for $\gamma\gamma \rightarrow \pi^0\pi^0$ where ChPT fails, whereas it simulates (good) results similar to the $\mathcal{L}\sigma\mathcal{M}$ for the above $a_{\pi\pi}^{(0)}$ scattering length.

To compare the $\mathcal{L}\sigma\mathcal{M}$ prediction (40) or the similar ChPT result with data, one recalls the $\pi\pi$ scattering length found from K_{e4} decay [46]

$$a_{\pi\pi}^{(0)}|_{K_{e4}} = (0.27 \pm 0.04)m_\pi^{-1}, \quad (41)$$

or the $\pi\pi$ scattering length inferred from πN partial wave data [47]

$$a_{\pi\pi}^{(0)}|_{\pi N} = (0.27 \pm 0.03)m_\pi^{-1}. \quad (42)$$

On the other hand, the Weinberg- $\mathcal{L}\sigma\mathcal{M}$ soft-pion scattering length (39) can acquire a hard-pion correction $\Delta a_{\pi\pi}^{(0)}$ due to the resonance decay $f_0(980) \rightarrow \pi\pi$. This was initially computed in ref. [48] based on a $f_0 \rightarrow \pi\pi$ decay width $\Gamma = 24 \pm 8 \text{ MeV}$. Since this 1992 PDG decay width has increased [14] to $\Gamma = 37 \pm 7 \text{ MeV}$, the hard-pion scattering length correction is now (with $g_{f_0}^2 = 16\pi m_{f_0}^2 \Gamma / 3p \approx 1.27 \text{ GeV}^2$, $\xi = m_\pi^2/m_{f_0}^2 \approx 0.02$)

$$\Delta a_{\pi\pi}^{(0)} = \frac{g_{f_0}^2}{32\pi m_\pi m_{f_0}^2} \left[\frac{5 - 8\xi}{1 - 4\xi} \right] \approx 0.07 m_\pi^{-1}. \quad (43)$$

Thus the entire Weinberg- $\mathcal{L}\sigma\mathcal{M}$ hard-pion correction prediction for the $I = 0$ s-wave $\pi\pi$ scattering length from (40) and (43) is

$$a_{\pi\pi}^{(0)} \approx 0.20 m_\pi^{-1} + 0.07 m_\pi^{-1} = 0.27 m_\pi^{-1}. \quad (44)$$

We note that this $a_{\pi\pi}^{(0)}$ $\mathcal{L}\sigma\mathcal{M}$ prediction (44) is in exact agreement with the central value of the data in (41) or (42).

VII. SUMMARY

In this paper we have studied low energy pion process and compared the data with the predictions of two chiral theories: (a) the chiral quark model (CQM) and its dynamically generated extension to the quark-level linear σ model ($L\sigma M$); (b) modern chiral perturbation theory (ChPT). We began in Sec.II by showing the direct link between the CQM-the quark-level $L\sigma M$ - and the Goldberger-Treiman relation (GTR), the $Z = 0$ condition and vector meson dominance (VMD), and the KSFRF relation. In Sec.III we used this CQM- $L\sigma M$ theory to compute the pion charge radius $r_\pi = \sqrt{3}/2\pi f_\pi \approx 0.61 \text{ fm}$ in the chiral limit. This agrees well with the observed [22] and VMD values $r_\pi = \sqrt{6}/m_\rho \approx 0.63 \text{ fm}$. In fact setting $r_\pi^{L\sigma M} = r_\pi^{VMD}$ leads to the rho-pion coupling $g_{\rho\pi\pi} = 2\pi$, which is only 2% greater than the observed PDG value [14]. On the other hand, ChPT fits the measured r_π to the parameter L_9 while maintaining that chiral log corrections are small [24].

Then in Sec.IV we computed the $\pi^0 \rightarrow \gamma\gamma$ and $\pi^+ \rightarrow e^+\nu\gamma$ amplitudes in the $L\sigma M$ and found both match data with the latter predicting $\gamma = F_A/F_V = 2/3$, while experiment gives [14] $\gamma = 0.68 \pm 0.34$. We extended the latter $L\sigma M$ loop analysis to charged pion polarizabilities in Sec.V, finding $\alpha_{\pi^+} = \alpha(12\pi^2 m_\pi f_\pi^2)^{-1} \approx 3.9 \times 10^{-4} \text{ fm}^3$, midway between the observed values. Also we studied $\gamma\gamma \rightarrow \pi^0\pi^0$ scattering at low energy, where data requires an s-wave cross section $\sigma < 10nb$ around energy $E \sim 700 \text{ MeV}$, and where ChPT predicts $\sigma \sim 20nb$. In contrast, refs [35,40], accounting for the ($L\sigma M$) scalar resonance $\varepsilon(700)$ appeared to predict $\sigma < 10nb$ five years before the first Crystal Ball data was published [37]. Finally, in Sec VI we extended Weinberg's soft pion (PCAC) prediction [10] for $a_{\pi\pi}^{(0)}$, the $I = 0$ s-wave $\pi\pi$ scattering length, to the (tree level) $L\sigma M$. Also, hard-pion corrections due to the $f_0(980) \rightarrow \pi\pi$ scalar resonance decays led to an overall scattering length $a_{\pi\pi}^{(0)} \approx 0.27m_\pi^{-1}$ in the extended $L\sigma M$, in perfect agreement with the central value of both the K_{l4} and πN -based measurements [46, 47] of $a_{\pi\pi}^{(0)}$.

In all of the above cases we compared these CQM- $L\sigma M$ predictions (depending upon *no* arbitrary parameters) with the predictions of chiral perturbation theory (ChPT depending on ten parameters $L_1 - L_{10}$) and found the latter theory almost always lacking. These results were tabulated in the following Table 1.

It is important to stress that a $Z = 0$ condition [18] is automatically satisfied in the strong interaction CQM- $L\sigma M$ -VMD-KSFRF theory and always “shrinks” quark loop graphs to “trees” for strong interaction processes such as $g_{\rho\pi\pi} = g_\rho$ or $\pi\pi \rightarrow \pi\pi$ and its accompanying $a_{\pi\pi}^{(0)}$ scattering length. This also makes VMD a tree-level phenomenology, as long stressed by Sakurai [16,19]. For processes involving a photon, however, such as for r_π , $\pi^0 \rightarrow 2\gamma$, $\pi^+ \rightarrow e^+\nu\gamma$, $\gamma\gamma \rightarrow \pi\pi$ and for α_π, β_π , the above $L\sigma M$ loop graphs must be considered (since a $Z = 0$ condition no longer applies).

In all of the above pion processes, the (internal) scalar σ meson plays an important role in ensuring the overall chiral symmetry (and current algebra-PCAC in the case of KSFRF, $\pi\pi \rightarrow \pi\pi$, $A_1 \rightarrow 3\pi$ and $\gamma\gamma \rightarrow \pi\pi$) for the relevant Feynman amplitude. Cases in point are $A_1 \rightarrow \pi(\pi\pi)_{\text{s wave}}$, and $\gamma\gamma \rightarrow \pi^0\pi^0$, where the internal σ mesons in Figs. 8b and 9b ensure the soft pion theorems (SPT), eqs. (33) and (34), which in fact are compatible with the data. This SPT role of the $\sigma(700)$ in $\gamma\gamma \rightarrow \pi^0\pi^0$ may explain why the ChPT approach does not conform to Crystal Ball data in Figs. (3,5) in refs [5,7], respectively [49].

In fact clues of a broad $\sigma(700)$ have been seen in (at least) seven different experimental analyses in the past 16 years [50]. As noted in ref.[20], the ChPT attempt to rule out a $L\sigma M$ structure was based on problems of a pure meson $L\sigma M$ (with that we agree)-but a pure meson $L\sigma M$ is *not* the CQM- $L\sigma M$ to which we adhere. The latter always begins in the chiral limit with axial current conservation due to a *quark*-level Goldberger-Treiman relation (GTR) eq. (1), which dynamically induces the $L\sigma M$ [2] starting from (CQM) *quark* loops.

As our final observation, the GTR-VMD-KSFRF basis of our proposed CQM- $L\sigma M$ theory are really examples [44] of soft-pion theorems and PCAC coupled to current algebra as used throughout the 1960's. Staunch advocates of ChPT in ref. [51] refer to such 1960 soft pion theorems as “low energy guesses” [LEG]. Instead they prefer the strict “low energy theorems” [LET] of modern ChPT. However ref. [51] concludes with an interesting remark: “it may be one of nature's follies that experiments seem to favour the original LEG over the correct LET”. Moreover ref. [6] concludes by noting that the VMD approach appears to give more reliable predictions for r_π , γ , α_π and $a_{\pi\pi}^{(0)}$ than does ChPT. We agree with both of these statements.

The author appreciates discussions with A. Bramon, S. Coon, R. Delbourgo, V. Elias, N. Fuchs, A. Kaloshin and R. Tarrach.

Figure Captions

Fig. 1 Quark loops for axial current $\langle 0 | \bar{q} \frac{1}{2} \lambda^3 \gamma_\mu \gamma_5 q | \pi^0 \rangle = i f_\pi q_\mu$.

Fig. 2 Quark loops for $\rho^0 \rightarrow \pi^+ \pi^-$.

Fig. 3 Quark loops for r_π .

Fig. 4 Quark loops for $\rho^0 \rightarrow \gamma$ em vector current.

Fig. 5 Quark loops for $\pi^0 \rightarrow \gamma\gamma$ decay.

Fig. 6 Quark (a) and meson (b) loops for $\pi^+ \rightarrow e^+ \nu \gamma$ in the L σ M.

Fig. 7 Plots of Crystal Ball $\gamma\gamma \rightarrow \pi^0 \pi^0$ data verses (a) the ChPT prediction; (b) the Kaloshin-Serebryakov prediction accounting for the $\varepsilon(700)$ scalar meson.

Fig. 8 Quark box (a) and triangle (b) graphs for $A_1 \rightarrow \pi(\pi\pi)_{s \text{ wave}}$ decay.

Fig. 9 Quark box (a) and triangle (b) graphs for $\gamma\gamma \rightarrow \pi^0 \pi^0$ scattering.

Table I

Comparison of chiral theory predictions of pion processes with experiment

	<u>CQM-LσM</u>	<u>ChPT to one loop</u>	<u>Data</u>
r_π	$0.61 fm$	$\frac{12}{f_\pi^2} L_9 + \text{chiral loops}$	$(0.63 \pm 0.01) fm$
$g_{\rho\pi\pi}$	$2\pi \approx 6.28$	$?$	6.1 ± 0.1
$F_{\pi\gamma\gamma}$	$\alpha/\pi f_\pi \approx 0.0258 GeV^{-1}$	$?$	$(0.026 \pm 0.001) GeV^{-1}$
$\gamma = \frac{F_A(0)}{F_V(0)}$	$\frac{2}{3}$	$32\pi^2(L_9 + L_{10})$	0.68 ± 0.34
α_{π^+}	$3.9 \times 10^{-4} fm^3$	$(\frac{4\alpha}{m_\pi f_\pi^2})(L_9 + L_{10})$	$(2.2 \text{ to } 6.8) \times 10^{-4} fm^3$
$\gamma\gamma \rightarrow \pi^0\pi^0$	$\sigma(E \sim 0.7 GeV) \sim 7nb$ and falling	$\sigma(E \sim 0.7 GeV) \approx 20nb$ and rising	$\sigma(E \sim 0.7 GeV) < 10nb$ and falling
$a_{\pi\pi}^{(0)}$	$0.27 m_\pi^{-1}$	$0.20 m_\pi^{-1}$	$(0.27 \pm 0.04) m_\pi^{-1}$

-
- [1] S. Weinberg, Phys. Rev. Lett. **65**, 1177 (1990). T. Hakioglu and M.D. Scadron, Phys. Rev. D**42**, 941 (1990); D**43**, 2439 (1991); M. D. Scadron, Mod. Phys. Lett. A **7**, 497 (1992); Phys. At. Nucl. **56**, 1595 (1993); P. Ko and S. Rudaz, Phys. Rev. D **50**, 6877 (1994).
 - [2] R. Delbourgo and M.D. Scadron, Mod. Phys. Lett. A**10**, 251 (1995); R. Delbourgo, A.A. Rawlinson and M.D. Scadron, ibid. A**13**, 1893 (1998) and hep-ph/9807505.
 - [3] J. Gasser and H. Leutwyler, Ann. Phys. (NY)**158**, 142 (1984). See Appendix II.
 - [4] J. Gasser and H. Leutwyler, Nucl. Phys. B**250**, 465 (1985); B**250**, 517 (1985).
 - [5] H. Leutwyler, Proc. XXVI Int. Conf. on High Energy Physics, Dallas, Aug. 1992, ed. J. R. Sanford, AIP Conf. Proc. No 272 (1993) p. 185.
 - [6] J. F. Donoghue and B. R. Holstein, Phys. Rev. D**40** 2378 (1989). B. R. Holstein, Comments Nucl. Part. Phys. **19**, 221 (1990).
 - [7] J. F. Donoghue and B. R. Holstein, Phys. Rev. D**48**, 137(1993).
 - [8] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966). More recently see A. Bramon, Riazuddin and M.D. Scadron, J. Phys. G**23**, 1065 (1998) and hep-ph/9709274.
 - [9] M. V. Terent'ev, Sov. J. Nucl. Phys. **16** 162 (1973).
 - [10] S. Weinberg, Phys. Rev. Lett. **17**, 616(1966).
 - [11] S. A. Coon and M. D. Scadron, Phys. Rev. C**23**, 1150(1981); M. D. Scadron, Repts. Prog. Phys. **44**, 213 (1981) find $1-f_{\pi}^{CL}/f_{\pi} = m_{\pi}^2/8\pi^2 f_{\pi}^2 \approx 0.03$.
 - [12] V. Elias and M.D. Scadron, Phys. Rev. D**30**, 647 (1984); V. Elias, M.D. Scadron and R. Tarrach, Phys. Lett. B**162**, 176 (1985).
 - [13] D. Bugg, A. Carter and J. Carter, Phys. Lett. B**44**, 278(1973); R. Koch and E. Pietarinen, Nucl. Phys. A**336**, 331 (1980); G Höhler, π N Newsletter **3**, 66 (1991).
 - [14] Particle Data Group, C. Caso et al. Eur. Phys. J. C**3**, 1(1998).
 - [15] Y. Nambu and G. Jona-Lasinio, (NJL) Phys. Rev. **122**, 345 (1961).
 - [16] J. J. Sakurai, Ann. Phys. NY **11**, 1 (1960).
 - [17] M. Gell-Mann, D. Sharp and W. Wagner, Phys. Rev. Lett. **8**, 261 (1962).
 - [18] A. Salam, Nuovo Cim. **25**, 224 (1962); S. Weinberg, Phys. Rev. **130**, 776 (1963). More recently see M.D. Scadron, Phys. Rev. D**57**, 5307 (1998) and hep-ph/9712425.
 - [19] J. J. Sakurai, Phys. Rev. Lett. **17**, 555 (1966).
 - [20] In Appendix II of Ref. [3] it is shown that a pure meson “Linear σ Model” is incompatible with data. While this may be true, such a pure meson model is *not* related to the above CQM-L σ M which is based on $m_{\pi} = 0$ and the *quark*-level GTR. Specifically the *quark* loop log divergent gap equ. (5) linking the L σ M to VMD in (6) cannot be obtained from a pure meson “L σ M”.
 - [21] R. Tarrach, Z. Phys. C**2**, 221 (1979); S. B. Gerasimov, Sov. J. Nucl. Phys. **29**, 259 (1979); V. Bernard, B. Hillar and W. Weise, Phys. Lett. B **205**, 16 (1988).
 - [22] A. F. Grashin and M. V. Lepeshkin, Phys. Lett. B**146**, 11(1984) reanalyze prior data suggesting $r_{\pi} \approx 0.66$ fm and find instead $r_{\pi} = 0.63 \pm 0.01$ fm.
 - [23] L. H. Chan, Phys. Rev. Lett. **39** 1125 (1977); *ibid* **55** 21 (1985); V. Novozhilov, Phys. Lett. B**228** 240(1989).
 - [24] Original chiral perturbation schemes of the 1970's as reviewed by H. Pagels, Phys. Repts. **16**, 219 (1975); J. Gasser and H. Leutwyler, *ibid* **87**, 77 (1982), claimed that the pion charge radius is scaled to a chiral log such that $r_{\pi} \rightarrow \infty$ as $m_{\pi} \rightarrow 0$, in contradiction with the CQM-L σ M, VMD, and data but compatible with the modern version of ChPT. See N. H. Fuchs and M. D. Scadron, J. Phys. G **12**, 1315 (1986) for an alternative point of view.
 - [25] J. Steinberger, Phys. Rev. **76**, 1180 (1949); S. Adler, Phys. Rev. **177** 2426 (1969); J. Bell and R. Jackiw, Nuovo Cim. **60**, 47 (1969).
 - [26] See e.g. B. de Witt and J. Smith, Field Theory in Particle Physics (North Holland, 1986) Vol. 1, pp. 284-290; A. S. Deakin, V. Elias and M. D. Scadron, Mod. Phys. Lett. A**9**, 955(1994).
 - [27] The CVC prediction (17) was first obtained by V. Vaks and B. Ioffe, Nuovo Cimento **10**, 342 (1958).
 - [28] N. Paver and M. D. Scadron, Nuovo Cim., **78A** 159(1983); Ll. Ametller, C. Ayala and A. Bramon, Phys. Rev. D**29**, 916(1984).
 - [29] P. Pascual and R. Tarrach, Nucl. Phys. B**146**, 520(1978); A. Bramon and M. D. Scadron, EuroPhys. Lett. **19**, 663 (1992).
 - [30] An alternative (alt) definition of $\alpha_{\pi}, \beta_{\pi}$ eliminates the 4π factor in eq. (23), meaning that α_{π}^{alt} is 4π larger than e.g. $\alpha_{\pi+}$ in (24)-(26), then measured in units 10^{-3} fm³.
 - [31] Y. M. Antipov et al., Phys. Lett. B**121**, 445(1983); Z. Phys. C**26**, 495(1985).
 - [32] T. A. Aibergenov, et al., Czech. J. Phys. B**36**, 948 (1986).
 - [33] D. Babusci et al., Phys. Lett. B**277**, 158 (1992).

- [34] A. I. L'vov, Sov. J. Nucl. Phys. **34**, 289 (1981).
- [35] A. E. Kaloshin and V. V. Serebryakov, Phys. Lett. B**278**, 198(1992), with $(\alpha - \beta)_\pi$ and $(\alpha + \beta)_\pi$ respectively extracted from data in Z. Phys. C **64**,689 (1994) and in hep-ph/942220 to appear in Yad. Fiz. These authors use polarizability units 4π times ours.
- [36] Mark II Collaboration, J. Boyer et al., Phys. Rev. D**42**, 1350 (1990).
- [37] Crystal Ball Collaboration, H. Marsiske et al., Phys. Rev. D **41**, 3324(1990); J. K. Bienlein report at 9th Int. Workshop on $\gamma\gamma$ collisions, La Jolla (1992).
- [38] D. Morgan and M. R. Pennington, Phys. Lett. B**272**, 134 (1991).
- [39] J. Bijnens and F. Cornet, Nucl. Phys. B**296**, 557 (1988); J. F. Donoghue, B. R. Holstein and Y.-C. R. Lin, Phys. Rev. D**37**, 2423 (1988).
- [40] A. E. Kaloshin and V. V. Serebryakov, Z. Phys. C**32**, 279 (1986).
- [41] A. N. Ivanov, M. Nagy and M. D. Scadron, Phys. Lett. B**273**, 137 (1991).
- [42] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
- [43] V. DeAlfaro, S. Fubini, G. Furlan and C. Rossetti, "Currents in Hadron Physics", North Holland Publ. (1973) Amsterdam, Chap. 5.
- [44] Also note that just as the $I = 1$ ρ pole in the $q^2 \rightarrow 0$ limit $g_{\rho\pi\pi}^2/m_\rho^2$ simulates the current algebra-VMD-KSRF factor $1/2f_\pi^2$ for $\pi\pi$ scattering, the $I = 0$ σ pole in the L σ M quartic limit λ/m_σ^2 simulates the current algebra-Weinberg factor $1/2f_\pi^2$.
- [45] J. Gasser and H. Leutwyler, Phys. Lett. B **125**, 321, 325(1983).
- [46] See, L. Rosselet et al., Phys. Rev. D**15**, 574 (1977); A. A. Belkov and S. A. Bunyatov, Sov. J. Nucl. Phys. **29**, 666 (1979), **33**, 410 (1981) for extractions of $a_{\pi\pi}^{(0)}$ from K_{l4} and $\pi N \rightarrow \pi\pi N$ data. M.M. Nagels et al. Nucl. Phys. B**147**, 189(1979) quotes Rosset's K_{e4} results giving $a_{\pi\pi}^{(0)}=0.28\pm0.05$ and also tables Manner's value 0.26 ± 0.05 .
- [47] R. Müller, G. E. Hite and R. J. Jacob, Z. Phys. C**8**, 199 (1981) and references therein concerning the extraction of $a_{\pi\pi}^{(0)}$ from $\pi N \rightarrow \pi N$ data.
- [48] R. J. Jacob and M. D. Scadron, Phys. Rev. D**25**, 3037 (1982).
- [49] A very recent ChPT calculation of $\gamma\gamma \rightarrow \pi^0\pi^0$ in two-loop order in J. Gasser et.al., Nucl. Phys. B**423**, 80(1994) finds better agreement with the Crystal Ball data at energies < 1 GeV.
- [50] A broad $\sigma(700)$ has been inferred from the data analyses of P. Estabrooks, Phys. Rev. D**19**, 2678 (1979); N. Biswas et al., Phys. Rev. Lett. **47**, 1378 (1981); T. Akesson et al., Phys. Lett. B **133**, 241 (1983); N. Cason et al., Phys. Rev. D **28**, 1586 (1983); A. Courau et al., Nucl. Phys. B **271**, 1 (1986). A broad σ but at lower mass (414 ± 20) MeV was inferred by the DM2 Collab., J. Augustin et al, Nucl. Phys. B**320**, 1 (1989). The PDG in ref. [14] now lists a broad σ (400-1200).
- [51] G. Ecker and Ulf-G. Meissner, "What is a Low-Energy Theorem?" HEP-PH-9409442, Sept. 1994.

Fig. 1



Fig. 2

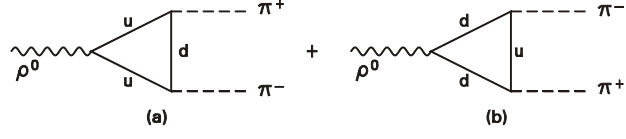


Fig. 3

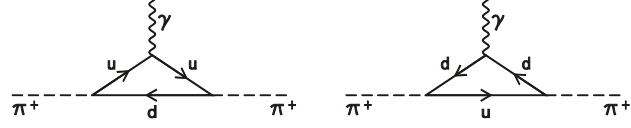


Fig. 4

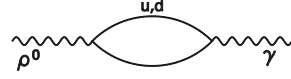


Fig. 5

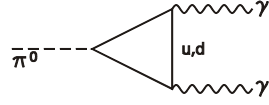


Fig. 6

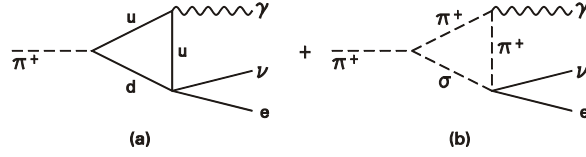


Fig. 7

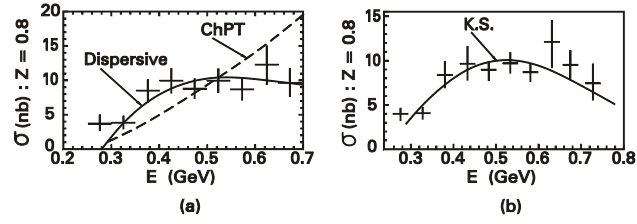


Fig. 8

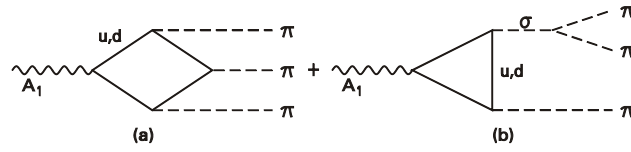


Fig. 9

